What Can the Bohr–Sommerfeld Model Show Students of Chemistry in the 21st Century?

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Bohr’s model of the atom (1) is generally considered to be important by both high school and university-level introductory (first-year) general chemistry textbooks. This model has also been the subject of textbook analyses based on a history and philosophy of science perspective (2–4). Bohr’s incorporation of Planck’s “quantum of action” to the classical electrodynamics of Maxwell represented a strange mixture for many of his contemporaries and philosophers of science (5). Margenau (6) referred to this inconsistent nature of Bohr’s model as a “Baroque tower on a Gothic base”. This shows how scientists, when faced with difficulties, often resort to such contradictory grafts. Despite the inconsistencies, the Bohr model of the atom helped scientists understand the paradoxical stability of Rutherford’s model and the hydrogen line spectrum.

Bohr’s model of the atom successfully explained the stability of atoms, the ionization energy, and the spectra of hydrogen-like ions (Balmer series), that is, those having a single electron (for example, He+, Li2+, and Be3+). Bohr’s first model claimed to predict all the lines in the hydrogen emission spectrum. However, experimental evidence indicated a hydrogen series (anomalous Pickering–Fowler ultraviolet series), where, according to Bohr, there should have been none (7). Bohr then proposed an alternative model of his research program, namely, the model of ionized helium (two protons orbited by an electron). The principal shortcomings of the Bohr model were that it could not explain the spectra of atoms containing more than one electron and the fine spectra into which spectral lines can be resolved using spectrographs of high resolving power (e.g., the Zeeman effect, a modification of atomic spectra by the application of a magnetic field). Sommerfeld (8, 9) presented a relativistic treatment to explain the fine structure of the hydrogen line spectrum, which was in fairly good agreement with observations and was considered to be a major success of Bohr’s conceptualization of the older quantum theory (10).

On the basis of these considerations, this study has the following objectives:

- Elaboration of a framework based on history and philosophy of science that led to the postulation of the Bohr–Sommerfeld model of the atom
- Formulation of suggestions for facilitating students’ understanding of models in chemistry

A History and Philosophy of Science Framework

Nature of Science

Considerable debate has emerged in the science education literature with respect to what the nature of science (NOS) is and how it can be included in the classroom. Undoubtedly, philosophers of science have debated this topic and continue to do so; for a recent example, see Giere (11). Furthermore, it cannot be denied that it is difficult for philosophers of science to achieve consensus in some areas, such as realism and antirealism concerning the entities treated by scientific theories. Nevertheless, science educators have achieved a fair amount of consensus with respect to various aspects of nature of science and Smith et al. have concluded (12, p 1101):

[Dis]agreements about the NOS is in fact reminiscent of people who argue about teaching evolution because scientists do not agree about what evolution is and how it works when there is, of course, remarkable agreement about the fundamental concepts in evolutionary theory (although there is disagreement about some peripheral issues).

In a similar vein, Matthews (13) has suggested that philosophy is not far below the surface in any science classroom. At a most basic level, any text or scientific discussion will contain terms such as law, theory, model, explanation, cause, truth, knowledge, hypothesis, confirmation, observation, evidence, and idealization. Matthews concludes (13, p 169):

Philosophy begins when students and teachers slow down the science lesson and ask what the above terms mean ... what things can be known and how can we know them, and about what things actually exist in the world and the relations possible between them.

In this context, it is important to note that scientific laws are epistemological constructions and do not describe the behavior of actual bodies. For example, Galileo’s law of free fall, Newton’s laws, and gas laws all describe the behavior of ideal bodies that are abstractions from the evidence of experience and the laws are true only when a considerable number of disturbing factors are eliminated. Bohr’s research program is an example of how scientists progress from simple to complex models, that is, from a fixed proton–nucleus in a circular orbit to elliptical orbits and so on (for details see Niaz, ref 14).

These are thought-provoking ideas provided that we want to foster conceptual understanding and not merely regurgitation
of experimental details. Interestingly, most science educators would agree that, of the various NOS aspects, perhaps one of the most important is precisely the tentative nature of scientific knowledge (15, 16).

**Tentative Nature of Scientific Knowledge**

The history of the structure of the atom since the late 19th and early 20th century shows that the models of J. J. Thomson (17), E. Rutherford (18), and N. Bohr (1) evolved in quick succession and had to contend with competing models based on rival research programs. This period of the history of structure of the atom has been the subject of considerable debate and controversy in the history and philosophy of science literature (5, 7, 10, 19–24). It is important to emphasize that scientific models increase in their heuristic and explanatory power (7). In other words, Rutherford’s model provided greater explanatory power as compared to Thomson’s model, which does not mean that Thomson was wrong. Similarly, Bohr’s model provided greater explanatory power as compared to Rutherford’s model, which does not mean that Rutherford was wrong. This precisely shows the tentative nature of scientific knowledge and its importance has been recognized for science education (15, 25). Similarly, if Einstein’s theory of gravitation is superseded by a version of string theory, it does not mean that Einstein was wrong, but that its sphere of applicability has been better defined.

Burbules and Linn have explained this in cogent terms (26, p 232): “If there is one thing that the history of science proves, it is that all theories turn out to be more or less ‘wrong’ in the end.” The history of science shows that progress in science is not merely based on the accumulation of experimental data (empiricist framework), but rather dependent on the creative imagination of the scientific community and rational arguments (for details see Niaz, ref 14). This sets the stage for going beyond Bohr’s model of the atom and understanding the contribution of Sommerfeld, which led to the Bohr—Sommerfeld model.

**Sommerfeld’s Elliptical Orbits**

As early as 1891, Michelson had reported that the Balmer series of the hydrogen spectrum was not composed of truly single lines (27). Although this was incompatible with Bohr’s theory, it was either ignored or not considered as a weighty argument because of the small order of magnitude involved (10). Sommerfeld (8, 9), however, considered Bohr’s analysis of the hydrogen spectrum as only approximate as it was based on only one quantum condition, the quantization of the angular momentum. Bohr’s orbits were all in a plane, which was too simple an assumption. Bohr himself also recognized that the original quantum theory was incomplete in the sense that, although it prescribed frequencies, it had nothing to say about intensities and polarizations (28). In contrast, Sommerfeld specified not only the shape of the electron’s orbit (which by analogy with planets in the solar system, could be elliptical instead of circular), but also its orientation in space (29). Contrary to Bohr’s 1913 picture, the electrons now moved in Keplerian ellipses, and during their orbits, they penetrated the region of internal electrons, thereby causing a coupling of the revolving electrons (30). In other words, the Bohr—Sommerfeld model considered the two-dimensional motion of the electron in its orbital plane. Treating the problem relativistically, Sommerfeld showed that (10, p 94):

[As in the case of every periodic motion under the influence of a central force, the electron with rest mass \( m \) describes a rosette or, more precisely, an ellipse with a slowly precessing perihelion and with one of its foci at the nucleus.]

On the basis of this basic idea of elliptical orbits, the Bohr—Sommerfeld model of the atom was widely accepted by the scientific community as an alternative to Bohr’s model. For example, Paschen’s (31) measurement of the helium spectrum was in agreement with Sommerfeld’s prediction. Later Sommerfeld developed these ideas further in his famous *Atombau und Spektrallinien* (32), which was written mainly for students and nonexperts in atomic physics, based on courses given at the University of Munich in 1916–1917, and first published in 1919. It ran through several new editions and was considered to be a “bible” by atomic theory physicists.

**Role of Auxiliary Hypotheses**

The success of the Bohr—Sommerfeld model was limited, as it could not explain many observed spectral lines. For every success of the model, there was a failure or an anomaly (28). These difficulties were resolved by Pauli’s exclusion principle, by placing the Bohr—Sommerfeld model on a more solid footing. Lakatos has explained this in cogent terms (7, p 153):

When some curious gaps appeared in Sommerfeld’s sophisticated models (some predicted lines never did appear), Pauli proposed a deep auxiliary hypothesis (his “exclusion principle”) which accounted not only for the known gaps but reshaped the shell theory of the periodic system of elements and anticipated facts then unknown.

The role of auxiliary hypotheses is important in philosophy of science (33). According to Lakatos (7), the appearance of empirical evidence contrary to the predictions of a theory does not immediately refute a theory. On the contrary, scientists try to introduce auxiliary hypotheses in order to protect the “hard core” of their theoretical formulations. In this context, Pauli’s exclusion principle was an attempt to avoid refutation of the Bohr—Sommerfeld model of the atom. The role of auxiliary hypotheses has also been recognized by Popper, provided that the auxiliary hypotheses increase the degree of falsifiability of the theory (cf. 34, p 153).

This contrasts with the textbook presentations, which generally argue that empirical evidence can unambiguously refute a theory. At this stage we would like to go a step further and suggest that, if Pauli’s exclusion hypothesis were an auxiliary hypothesis that tried to “protect” the Bohr—Sommerfeld model, then by analogy Sommerfeld’s elliptical orbits also played a similar role by avoiding the refutation of Bohr’s model. This again shows that scientists not only keep the refuting evidence in abeyance, but also try to put forward new hypotheses (i.e., auxiliary hypotheses) to protect an existing theory with some explanatory potential.

Let us see how Lakatos conceptualizes progress in science (7, p 150):

> It is interesting that just as Einstein got worried and slowed down in the middle of the spectacular progress of quantum physics by 1913, Bohr got worried and slowed down by 1916; and just as Bohr had, by 1913, taken the initiative from Einstein, Sommerfeld had taken the initiative from Bohr by 1916.
Understanding the Nature of Science

Considerable research has been conducted to show that students’ understanding of how scientists work is far from satisfactory (15, 16, 35–43). Although a detailed review of research on the subject is beyond the scope of this article, results from one study are reported here. Abd-El-Khalick (44), in a study based on 153 undergraduate and graduate students at a west coast university in the United States, found that the majority of the students held naïve views or inaccurate understanding of the following aspects of the nature of science:

- The tentative, empirical, inferential, theory-laden, imaginative, and creative nature of scientific knowledge
- Social and cultural factors in theory change
- Role of theory and prior expectations in designing and conducting experiments
- Hierarchical view of the relationship between theories and laws
- Use of “the scientific method”, that is, students incorrectly think that science is characterized by the scientific method (belief that there is a recipe, like a stepwise procedure that all scientists follow when they do science, see ref 15)

According to the author, these results are consistent with ones reported in a plethora of studies.

In a recent study, Abd-El-Khalick, Waters, and Le (45) have drawn attention to the importance of including NOS characterizations in high school chemistry textbooks. These authors analyzed 14 textbooks, including five series spanning one to four decades, with respect to the following NOS aspects:

- Empirical, tentative, inferential, creative, theory-driven
- Myth of the scientific method
- Nature of scientific theories and laws
- Social and cultural embeddedness of science

Results from this study revealed that chemistry textbooks fared poorly in their representation of the NOS, which led the authors to conclude, “These trends are incommensurate with the discourse in national and international science education reform documents” (45, p 835).

Similarly, Páez and Niaz (46) have analyzed 16 high school chemistry textbooks published in Venezuela to determine the degree to which these textbooks coincide with the present-day viewpoint of educational reform based on a history and philosophy of science perspective. These authors reported that a majority of the textbooks reflect the positivist perspective and ignore important aspects of the nature of science, such as:

- Scientific knowledge is tentative.
- The scientific method is a myth.
- A dichotomy exists between laws and theories.
- Heuristic principles have a role.
- Observations are theory-laden.
- Conflicts, rivalries, and controversies play an important part in scientific development.

Suggestions for Facilitating Students’ Understanding of Atomic Models in Chemistry

The Bohr model of the atom had a major drawback because it could not explain the spectra of He	extsuperscript{\textsuperscript{+}} and Li	extsuperscript{\textsuperscript{2\textsuperscript{+}}}. It could not explain the spectra of even the alkali metals (Li, Na, K, Rb, Cs), which have a single valence electron. To facilitate students’ understanding of models in chemistry, Dickerson et al. (47) have included the Bohr–Sommerfeld model of the atom in a section entitled, Need for a Better Theory, along with a figure of the elliptical orbits; following are some of the excerpts (47, pp 269–271):

Arnold Sommerfeld (1868–1951) proposed an ingenious way of saving the Bohr theory. He suggested that orbits might be elliptical as well as circular. Furthermore, he explained the differences in stability of levels with the same principal quantum number, \( n \), in terms of the ability of the highly elliptical orbits to bring the electron closer to the nucleus (Figure 7–15). For a point nucleus of charge +1 in hydrogen, the energies of all levels with the same \( n \) would be identical. But for a nucleus of +3 screened by an inner shell of two electrons in Li, an electron in an outer circular orbit would experience a net attraction of +1, whereas one in a highly elliptical orbit would penetrate the screening shell and feel a charge approaching +3 for part of its traverse. Thus, the highly elliptical orbits would have the additional stability. ... The \( j \) orbit, being the most elliptical of all in Sommerfeld’s model, would be much more stable than the others in the set of common \( n \).... The Sommerfeld scheme led no further than the alkali metals. Again an impasse was reached, and an entirely fresh approach was needed.

It is important to note that the shapes of Sommerfeld’s ellipses are significantly different from orbitals in wave mechanics. An important feature of this model was that the \( j \) orbits become highly elliptical as the \( n \) quantum number increases, thus, providing additional stability (see Figure 1). It is important to note how Dickerson et al. (47) first point out a “fatal weakness” in the Bohr model of the atom, followed by Sommerfeld’s “ingenious way” of saving it and finally refer to an “impasse” requiring a new approach. This presents the tentative nature of scientific models in a way that can appeal to the students. Furthermore, very few general chemistry textbooks mention the Bohr–Sommerfeld model and much less refer to the tentative nature of models.

From a historical and pedagogical perspective, this study provides the following suggestions for facilitating students’ understanding of atomic models:

- Bohr’s model provided an explanation for the paradoxical stability of the Rutherford model and spectra of hydrogen-like ions.
- The Bohr–Sommerfeld model, though helpful in the beginning, needed improvements that were introduced by Pauli’s exclusion principle and others.
- For students it would be a surprise to know that, despite its popularity and novelty, Bohr’s model of the atom (a “hard core” theory) only explained the stability, ionization energy, and the spectra of hydrogen-like ions, that is, those possessing a single electron (H\textsuperscript{+}, Li\textsuperscript{2\textsuperscript{+}}, Be\textsuperscript{3\textsuperscript{+}}).
- Sommerfeld’s innovation (auxiliary hypothesis) by introducing elliptical orbits, helped to restore the viability of Bohr’s model to a certain degree.

Figure 1. Elliptical orbits in the Bohr–Sommerfeld model. (Electronically reproduced by permission of Pearson Education, Inc., Upper Saddle River, NJ, from Dickerson et al., 1984, Figure 7–15, p 270; see ref 47).
The Bohr—Sommerfeld's model went no further than the alkali metals, which led scientists to look for other models.

This example shows how scientific models are tentative in nature.

From a history and philosophy of science perspective, Bohr—Sommerfeld's model can be interpreted as an "auxiliary hypothesis" to save the "hard core" of Bohr's theory (cf. Lakatos, ref 7).

**Literature Cited**

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